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A possible explanation of sound conversion at the free surface of superfluid helium near T_λ

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Abstract. The critical behaviour of the transport coefficients of the free surface of liquid helium above T_λ is studied by using the renormalisation group results of the semi-infinite model E, and a possible explanation for the experiment of Wiechert and Buchholz on sound conversion at the free surface of He II is given by extrapolating these results to the region below T_λ.

1. Introduction

A few years ago, an interesting experimental study of the free surface of superfluid helium was performed by Wiechert and Buchholz (WB) [1] who measured the reflection, transmission and conversion coefficients of sound waves at the surface. While their data outside the critical region agree quantitatively with their own theory [2], there appeared a pronounced deviation of the data from the theory near the λ-point, T_λ. This phenomenon suggested that the Onsager coefficients describing the transport across the fluid–gas interface have a critical anomaly at T_λ.

It is well known that the critical behaviour of superfluid helium is described approximately by the dynamic phase-transition model, termed model E by Halperin and coworkers [3, 4]. This model was shown to be successful, at least qualitatively, in describing the thermal conductivity of the bulk. Therefore, we conclude that the surface of the system can be discussed by using the semi-infinite analogue of the model which has a free surface. The aim of the present paper is to use the semi-infinite model E to study the critical behaviour of the transport coefficients relating to the free surface of superfluid helium, and propose an explanation of the deviation between the theory and the experimental data of WB.

The outline of the paper is as follows. In § 2, we introduce the model and define the cumulants of the system. In § 3, we consider the renormalisation of the cumulants and solve the renormalisation group equations of the cumulants. In § 4 we discuss the properties of the fixed points and derive the critical behaviour of the transport coefficients relating to the surface in the region $\tau > 0$ ($\tau = T/T_\lambda - 1$). In § 5, using some assumptions,

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we give a correction to the theory of WB, and compare the corrected results with the experimental data of WB. Finally, a brief summary and conclusion is given in § 6.

2. The model

A semi-infinite model E is defined by the following equations

$$\partial\psi/\partial t = -\Gamma_0 \delta H/\delta\psi^* - \mathrm{i}g_0\psi\delta H/\delta m + \theta \tag{2.1a}$$

$$\partial m/\partial t = \nabla^2\lambda_0^m \delta H/\delta m + 2g_0 \mathrm{Im}(\psi^*\delta H/\delta\psi^*) + \xi \tag{2.1b}$$

$$H = H_0 - \int \mathrm{d}^{d-1}\boldsymbol{r}\int_0^\infty \mathrm{d}x_\perp\,[h_m m + \mathrm{Re}(h\psi^*)] - \int \mathrm{d}s\,[h_m^{\mathrm{s}} m_{\mathrm{s}} + \mathrm{Re}(h^{\mathrm{s}}\psi_{\mathrm{s}}^*)] \tag{2.2a}$$

$$H_0 = \tfrac{1}{2}\int \mathrm{d}^{d-1}\boldsymbol{r}\int_0^\infty \mathrm{d}x_\perp\,\left(r_0|\psi|^2 + |\nabla\psi|^2 + \frac{2u_0}{4!}|\psi|^4 + \tfrac{1}{2}m^2 + \tfrac{1}{2}c^{\mathrm{o}}|\psi|^2\delta(x_\perp)\right) \tag{2.2b}$$

where $\psi(\boldsymbol{x}, t)$ is a complex order parameter of the superfluid, and the $m(\boldsymbol{x}, t)$ is some linear combination of energy and mass densities, h_m and h are infinitesimal applied fields of the bulk, h_m^{s} and h^{s} are those of the surface. θ and ξ are the usual Langevin random forces [4]. In (2.2) c^{o} is the so-called surface enhancement characterising the surface free energy. After introducing two purely imaginary response fields $\hat{\psi}(\boldsymbol{x}, t)$ and $\hat{m}(\boldsymbol{x}, t)$, called MSR conjugate fields [5], the part probability density of the stochastic variables ψ and m can be written in the form

$$P\{\psi, \psi^*, m\} = \text{constant} \times \exp\{Q\} \tag{2.3}$$

$$\begin{aligned} Q[\psi, \hat{\psi}, m, \hat{m}] = \int \mathrm{d}^{d-1}\boldsymbol{r}\int_0^\infty \mathrm{d}x_\perp \int \mathrm{d}t\{&-\hat{\psi}^* 2\Gamma_0\hat{\psi} \\ &+ [\mathrm{i}\hat{\psi}(\partial\psi/\partial t + \Gamma_0 r_0\psi + c^{\mathrm{o}}\psi\Gamma_0\delta(x_\perp) - \Gamma_0\nabla^2\psi + (u_0/3!)\Gamma_0|\psi|^2\psi \\ &- \Gamma_0\delta(x_\perp)\partial x_\perp\psi - h\Gamma_0 - h^{\mathrm{s}}\Gamma_0\delta(x_\perp) + \mathrm{i}g_0\psi m \\ &- \mathrm{i}g_0\psi h_m) - \mathrm{cc}] + \hat{m}\lambda_0^m\nabla^2\hat{m} + \hat{m}\lambda_0^m\nabla^2 m \\ &+ \mathrm{i}\hat{m}[\partial m/\partial t + \lambda_0^m\nabla^2(h_m + h_m^{\mathrm{s}}\delta(x_\perp) - m) \\ &+ 2g_0\mathrm{Im}(\psi^*\psi c^{\mathrm{o}}\delta(x_\perp) - \psi^*\partial x_\perp\psi\delta(x_\perp) - \psi^*\nabla^2\psi) \\ &- 2g_0\mathrm{Im}(h\psi^* + h^{\mathrm{s}}\psi^*\delta(x_\perp))]\}. \end{aligned}$$

For the same reason as given in [6], the Jacobian term in (2.3) is eliminated. The surface terms in (2.3) imply the boundary conditions

$$\begin{aligned} \partial x_\perp\psi|_{x_\perp=0} &= c^{\mathrm{o}}\psi_{\mathrm{s}} \qquad & \partial x_\perp\hat{\psi}|_{x_\perp=0} &= c^{\mathrm{o}}\hat{\psi}_{\mathrm{s}} \\ \partial x_\perp\psi^*|_{x_\perp=0} &= c^{\mathrm{o}}\psi_{\mathrm{s}}^* \qquad & \partial x_\perp\hat{\psi}^*|_{x_\perp=0} &= c^{\mathrm{o}}\hat{\psi}_{\mathrm{s}}^* \end{aligned} \tag{2.4}$$

As a result of the boundary conditions the Gaussian part of Q can be diagonalised.

The generating functional for the connected correlation and the response functions are introduced as

$$Z[L, L_1, I, I_1] = \ln\left\{ \int D\{\mathrm{i}\hat{\psi}, \psi, \mathrm{i}\hat{m}, m\} \exp\left[Q + \int \mathrm{d}x\, \mathrm{d}t\, (L\,\psi + Im) + \int \mathrm{d}S\, (L_1 \psi_s + I_1 m_s) \right] \right\}. \tag{2.5}$$

The cumulants of the system can be derived as follows

$$\bar{W}_{\psi}^{(\hat{N},N,\hat{M},M)} = \prod_{j=1}^{\hat{N}} \frac{\delta}{\delta h(x_j, t_j)} \prod_{k=1}^{N} \frac{\delta}{\delta L(x_k, t_k)} \prod_{l=1}^{\hat{M}} \frac{\delta}{\delta h^s(r_l, t_l)} \prod_{m=1}^{M} \frac{\delta}{\delta L_1(r_m, t_m)} Z \Bigg|_{\substack{L=L_1=0 \\ h=h^s=0}}$$

$$= \left\langle \prod_{j=1}^{\hat{N}} [\Gamma_0 \hat{\psi} + \mathrm{i}g_0 \hat{m} \psi^*](x_j, t_j) \prod_{k=1}^{N} \psi(x_k, t_k) \times \prod_{l=1}^{\hat{M}} [\Gamma_0 \hat{\psi}_s + \mathrm{i}g_0 m_s \psi_s^*](r_l, t_l) \prod_{m=1}^{M} \psi_s(r_m, t_m) \right\rangle \tag{2.6}$$

$$\bar{W}_{m}^{(\hat{N},N,\hat{M},M)} = \prod_{j=1}^{\hat{N}} \frac{\delta}{\delta h_m(x_j, t_j)} \prod_{k=1}^{N} \frac{\delta}{\delta I(x_k, t_k)} \prod_{l=1}^{\hat{M}} \frac{\delta}{\delta h_m^s(r_l, t_l)} \prod_{m=1}^{M} \frac{\delta}{\delta I_1(r_m, t_m)} Z \Bigg|_{\substack{I=I_1=0 \\ h_m=h_m^s=0}}$$

$$= \left\langle \prod_{j=1}^{\hat{N}} [-\lambda_0^m \nabla^2 \hat{m} + \mathrm{i}g_0(\hat{\psi}\psi - \hat{\psi}^*\psi^*)](x_j, t_j) \prod_{k=1}^{N} m(x_k, t_k) \times \prod_{l=1}^{\hat{M}} [-\lambda_0^m \nabla^2 \hat{m}_s + \mathrm{i}g_0(\hat{\psi}_s \psi_s - \hat{\psi}_s^* \psi_s^*)](r_l, t_l) \prod_{m=1}^{M} m_s(r_m, t_m) \right\rangle. \tag{2.7}$$

For example, the linear response functions of the bulk are

$$R_{\psi,m}(x, t, x', t') = \bar{W}_{\psi,m}^{(1,1,0,0)}(x, t, x', t') \tag{2.8}$$

and the corresponding ones of the surface are

$$R_{\psi,m}(r, t, r', t') = \bar{W}_{\psi,m}^{(0,0,1,1)}(r, t, r', t'). \tag{2.9}$$

The surface–bulk linear response functions are the responses of the stochastic variables $\psi(x, t)$ and $m(x, t)$ of the surface to the time-dependent applied fields $h(x, t)$ and $h_m(x, t)$. They are defined as

$$R_{\psi,m}(x, t, r', t') = \bar{W}_{\psi,m}^{(1,0,0,1)}(x, t, r', t'). \tag{2.10}$$

It is convenient for the further discussion in momentum–frequency space, therefore we introduce the Fourier transformation in the same form as in [6], but we omit them for simplicity.

3. Renormalisation and renormalisation equations

Standard theory suggested that we may consider the perturbation expansion of the cumulants in the limit $\Lambda \to \infty$, and in the $d = 4 - \varepsilon$ space the UV-singularities can be absorbed into the renormalisation functions which should be introduced by considering the dimensional analysis and the counterterms of Q. The ones chosen to absorb the bulk UV-singularities are as follows

$$\begin{cases} \psi = Z_\psi^{1/2}\psi^R & \hat{\psi} = Z_{\hat{\psi}}^{1/2}\hat{\psi}^R \\ m = Z_m^{1/2} m^R & \hat{m} = Z_{\hat{m}}^{1/2}\hat{m}^R \\ \Gamma_0 = \mu^{-2} Z_r \Gamma & \lambda_0^m = \mu^{-2} Z_\lambda^{-1}\lambda^m \\ u_0 = \mu^\varepsilon S_d^{-1} Z_u u & g_0 = \mu^{\varepsilon/2} S_d^{-1} g Z_g \qquad S_d = [2^{d-1}\pi^{d/2}\Gamma(d/2)]^{-1} \\ r_0 = \mu^2 Z_\tau \tau \end{cases} \tag{3.1}$$

where Z_x ($x = \psi, \hat{\psi}, m, \hat{m}, u, \lambda, \Gamma, \tau, g$) are the bulk renormalisation factors. The additional primitive UV-divergents due to the surface should be absorbed by local surface counterterms; we therefore introduce the renormalisation factors Z_1, $\hat{Z}_1$, Z_2, $\hat{Z}_2$ and Z_c via [6, 7]

$$\begin{cases} \psi_s = (Z_\psi Z_1)^{1/2}\psi_s^R = Z_1^{1/2}(\psi^R)_s \\ \hat{\psi}_s = (Z_{\hat{\psi}}\hat{Z}_1)^{1/2}\hat{\psi}_s^R = \hat{Z}_1^{1/2}(\hat{\psi}^R)_s \\ m_s = (Z_m Z_2)^{1/2} m_s^R = Z_2^{1/2}(m^R)_s \\ \hat{m}_s = (Z_{\hat{m}}\hat{Z}_2)^{1/2}\hat{m}_s^R = \hat{Z}_2^{1/2}(\hat{m}^R)_s \\ c^\circ = \mu Z_c c. \end{cases} \tag{3.2}$$

For the reason that the surface cannot affect the critical properties of the bulk far away from the surface, we may conclude that the difference between a bulk renormalisation factor of the semi-infinite system and its analogue in the infinite system is a finite quantity of the order $O(\varepsilon^0)$. This conclusion has been proved exactly for the models A, B and C [6, 7], we therefore do not give an exact demonstration here. As a result of this conclusion, we can set $Z_x(x = \psi, \hat{\psi}, m, \hat{m}, u, \lambda, \Gamma, \tau, g)$ equal to their analogues in the infinite system [8].

The response and correlation functions satisfy the fluctuation-dissipation theorem

$$R_{\psi,m}(t) - R_{\psi,m}(-t) = \partial C_{\psi,m}(t)/\partial t. \tag{3.3}$$

It follows that the bare and renormalised responses are in the same relation as the correlation functions

$$R_m(-\mathrm{i}\omega/\lambda_0^m, \boldsymbol{x}_1, \boldsymbol{x}_2, u_0, r_0, c^\circ, \Gamma_0) = Z_m R_m^R(-\mathrm{i}\omega/\lambda^m, \boldsymbol{x}_1, \boldsymbol{x}_2, u, \tau, c, \Gamma). \tag{3.4}$$

For the surface cases it can be written

$$(R_m)_s(-\mathrm{i}\omega/\lambda_0^m, \boldsymbol{r}_1, \boldsymbol{r}_2, u_0, r_0, c^\circ, \Gamma_0) = Z_m Z_2 (R_m^R)_s(-\mathrm{i}\omega/\lambda^m, \boldsymbol{r}_1, \boldsymbol{r}_2, u, \tau, c, \Gamma). \tag{3.5}$$

It has been proved that the zero-frequency limit of the response functions are static correlation functions [8]. This allows us to identify the Z_1, Z_2 and Z_c as just the same functions that emerge from a renormalisation procedure applied to a purely static theory. The static limit of the model E is just the same as the model A, therefore, the Z_1 and Z_c can be read off from [7], and the factor Z_2 is given by its analogue in model C [6] with a parameter change $u_0 - 3\gamma_0^2 \rightarrow u_0$.

Now there are only two factors $\hat{Z}_1$ and $\hat{Z}_2$ keeping unknown, but they are not necessary because they do not appear in the RG equations. This simply means that the additional primitive divergence does not depend on the dynamic properties of the system.

The relations between the bare and the renormalised cumulants are the following:

$$[\bar{W}_m^{(\hat{N},N,\hat{M},M)}]_R = \mu^{-2(\hat{N}+\hat{M})} Z^{-(\hat{N}+N+\hat{M}+M/2} Z_2^{-(\hat{M}+M)/2} \bar{W}_m^{(\hat{N},N,\hat{M},M)}. \tag{3.6}$$

After defining two parameters

$$w = \lambda^m/\Gamma \qquad f = g^2(\lambda^m\Gamma) \tag{3.7}$$

we can write the RG equations in the form

$$\left[\mu\frac{\partial}{\partial\mu} + \beta_u\frac{\partial}{\partial u} + \sum_{l=\tau,c,w,f} W_l\frac{\partial}{\partial l} + \eta_\lambda\omega\frac{\partial}{\partial\omega} + (\hat{M}+M)\eta_2/2 - 2(\hat{M}+\hat{N})\right]$$
$$\times [\bar{W}_m^{(\hat{N},N,\hat{M},M)}(-\mathrm{i}\omega/\lambda^m, \boldsymbol{p}, k_1, k_2, \tau, u, c, w, f)]_R = 0 \tag{3.8}$$

where

$$\beta_u = \mu\frac{\partial}{\partial\mu}\bigg|_0 u \qquad W_l = \mu\frac{\partial}{\partial\mu}\bigg|_0 l \qquad (l = \tau, c, w, f)$$

$$\eta_x = \mu\frac{\partial}{\partial\mu}\ln Z_x, (x = \lambda, 2).$$

For the case $\mu = 1, |\boldsymbol{p}| = k_1 = k_2 \to 0$, and $\tau = 0$, these equations can be solved as

$$[\bar{W}_m^{(\hat{N},N,\hat{M},M)}(-\mathrm{i}\omega/\lambda^m, p, 0, u, c, w, f)]_R = p^{(d_E+\eta_B^*)}$$
$$\times [\bar{W}_m^{(\hat{N},N,\hat{M},M)}(-\mathrm{i}\omega/\lambda^m p^{-(2+\eta_\lambda^*)}, 1, 0, u^*, c^*, w^*, f^*]_R \tag{3.9}$$

where

$$d_E = \begin{cases} d(\hat{N}+N+\hat{M}+M)/2 + 2(\hat{M}+\hat{N}) - d & (N+\hat{N} \neq 0) \\ d(\hat{N}+N+\hat{M}+M)/2 + 2(\hat{M}+\hat{N}) - d + 1 & (N+\hat{N} = 0) \end{cases} \tag{3.10}$$

$$\eta_B = (\hat{M}+M/2)\eta_2 - 2(\hat{M}+\hat{N}). \tag{3.11}$$

The transport coefficient of the bulk is defined as

$$\kappa_\mathrm{b} = p^{-2}\left[\frac{\partial}{\partial(-\mathrm{i}\omega)}(\bar{W}_m^{(1,1,0,0)})^{-1}\right]_{\omega=0}^{-1}. \tag{3.12}$$

Accordingly, the other two transport coefficients related to the surface may be introduced in the same way. The corresponding one for the surface can be written in the form

$$\kappa_\mathrm{s} = p^{-2}\left[\frac{\partial}{\partial(-\mathrm{i}\omega)}(\bar{W}_m^{(0,0,1,1)})^{-1}\right]_{\omega=0}^{-1} \tag{3.13}$$

and the transport coefficient describing the transport between the bulk and the surface is

$$\kappa_\mathrm{bs} = p^{-2}\left[\frac{\partial}{\partial(-\mathrm{i}\omega)}(\bar{W}_m^{(1,0,0,1)})^{-1}\right]_{\omega=0}^{-1}. \tag{3.14}$$

Using (3.9) we can write

$$\left.\begin{aligned} &\kappa_\mathrm{b} \sim p^{\eta_\lambda^*} \\ &\kappa_\mathrm{s} \sim p^{1+\eta_\lambda^*+\eta_2^*} \\ &\kappa_\mathrm{bs} \sim p^{\eta_\lambda^*+\eta_2^*/2} \end{aligned}\right\} \quad (\omega = \tau = 0, p \to 0). \tag{3.15}$$

or alternatively

$$\left.\begin{aligned} &\kappa_{\mathrm{b}} \sim \tau^{\nu\eta^*_\lambda} \\ &\kappa_{\mathrm{s}} \sim \tau^{\nu(1+\eta^*_\lambda+\eta^*_2)} \\ &\kappa_{\mathrm{bs}} \sim \tau^{\nu(1+n^*_\lambda+\eta^*_2)/2} \end{aligned}\right\} \quad (\tau \to 0+, \omega = p = 0). \tag{3.16}$$

4. Fixed point and critical exponents

The physical point of helium in the (ε, n) plane lies at $\varepsilon = 1$ and $n = 2$. It means that the surface transition is the so-called ordinary transitions and the fixed point of c is infinite

$$c^* = \infty. \tag{4.1}$$

From [6] we know that

$$\eta^*_2 = 2(\eta^*_\tau - \eta^*_c) = (2/\nu)(1 - \Phi - \nu) \tag{4.2}$$

where

$$\Phi = \tfrac{1}{2} - [(n+2)/4(n+8)]\varepsilon - [(n+2)/(n+8)^3][8\pi^2(n+8) - (n^2+35n+156)]\varepsilon^2 + \mathrm{O}(\varepsilon^3) \tag{4.3}$$

is given in [7]. Because the surface does not affect the critical properties of the bulk far away from the surface, the bulk exponent η^*_λ takes the same value of its analogue of the infinite system [8]

$$\eta^*_\lambda = -\tfrac{1}{2}\varepsilon. \tag{4.4}$$

Equation (4.4) corresponds to the IR stable fixed point. Then, from (3.16), (4.3) and (4.4) we can write the asymptotic behaviour of the transport coefficients as follows:

$$\kappa_{\mathrm{b}} \sim \tau^{-0.33} \qquad \kappa_{\mathrm{s}} \sim \tau^{-0.34} \qquad \kappa_{\mathrm{bs}} \sim \tau^{-0.66}. \tag{4.5}$$

From the results given above we see that at the λ-point the transport coefficient on the surface diverges as strongly as that of the bulk, but that one between the bulk and surface diverges much more strongly than that of the bulk.

There is still another possible physical fixed point apart from the IR stable one, which is termed the weak-scaling fixed point [8]. If this fixed point is the physical one, the exponents of the transport coefficients will have some small corrections. In the present paper we do not consider the possibility of this point, since we only consider a qualitative correction to theoretical work of WB.

Of course, the asymptotic behaviour of the transport coefficients given by (4.5) is only an approximate one. A non-asymptotic treatment is necessary if one wants to consider the calculations quantitatively. For the same reason that we have just mentioned above, we do not take the careful calculations about the corrections of the model E. The reader is referred to references [11, 12] where the non-asymptotic treatments for the infinite models E and F are given.

5. Critical behaviour of the Onsager coefficients and the sound conversion below T_λ

Here we want to use the results of (4.5) to explain the experimental data of WB. First of all, we assume that below T_λ the critical behaviour of κ_{b}, κ_{s} and κ_{bs} are still described by equation (4.5). This is based on a well known fact that so far the experimental work shows that any thermodynamic quantity which is observable has the same critical exponent both above and below the critical point. Therefore, this property may be predicted to be held for all cases.

In the experiment of WB [1], a monochromatic longitudinal sound wave was normally incident from the vapour onto the plane separating the fluid and the vapour phases of helium. A deviation of the system from its equilibrium is caused by such an incidence, causing a mass and thermal current to emerge across the interface. According to Onsager's theory these currents may be formulated in the linear phenomenological equations:

$$J_M = L_{MM}X_M + L_{ME}X_E \tag{5.1}$$

$$J_E = L_{EM}X_M + L_{EE}X_E \tag{5.2}$$

where the L_{MM}, L_{ME}, L_{EM} and L_{EE} are Onsager coefficients and the forces X_M and X_E take the form [10]

$$X_M = \mu_o T_o^{-2}(\tilde{T}_g - \tilde{T}_l) + T_o^{-1}(\tilde{\mu}_l - \tilde{\mu}_g) \tag{5.3}$$

$$X_E = T_o^{-2}(\tilde{T}_l - \tilde{T}_g) \tag{5.4}$$

where the $\tilde{T}$ and $\tilde{\mu}$ are defined as the deviations of the temperature and the chemical potential from the equilibrium value:

$$\tilde{T} = T - T_o \qquad \tilde{\mu} = \mu - \mu_o$$

the subscript o refers to the equilibrium value and the indices l and g refer to the liquid and the gas, respectively. Near the T_λ-point the thermal conductivity between the surface and the bulk of the He II is strongly divergent, so we assume $\tilde{T}_l \simeq \tilde{T}_g$, then we have

$$J_M \simeq L_{MM}X_M \tag{5.5}$$

$$J_E \simeq L_{EM}X_M. \tag{5.6}$$

Consider that at T_λ the system is equivalent to a system having infinite surface enhancement. Hence, the evaporation of the He II can be neglected because the surface acts as a 2D infinite deep potential well, and J_M describes only the mass current caused by the condensation of the vapour on the surface, so that $L_{MM} \sim \kappa_s$, since the condensed mass is local to the surface. Accordingly, J_E describes only the transport of the latent heat of the condensate moving from the surface to the bulk of He II. Thus we can write that $L_{EM} \sim \kappa_{bs}$.

The incident sound wave will partly be reflected, partly transmitted as a first sound wave in the liquid, and partly transformed into a thermal wave travelling back into the vapour and a second sound wave in the liquid. We assume here that the thermal wave travelling back into the vapour is very weak for the reason that the thermal conductivity of He II is divergent at T_λ. With these assumptions we can regenerate the acoustic coefficients from (47)–(49) of [2]

$$R_{GG} = [1 + (K_1\pi' + K_2)Z_2L - K_1u_G]/[1 + (K_1\pi' + K_2)Z_2L + K_1u_G] \tag{5.7}$$

$$D_{G1} = 1 + R_{GG} \tag{5.8}$$

$$T_{G2} = 2K_1Z_2L/[1 + (K_1\pi' + K_2)Z_2L + K_1u_G] \tag{5.9}$$

where

$$K_1 = L_{MM}/(\rho_{Go}T_o) \tag{5.10}$$

$$K_2 = (L_{EM} - L_{MM}h_{Go})/T_o^2. \tag{5.11}$$

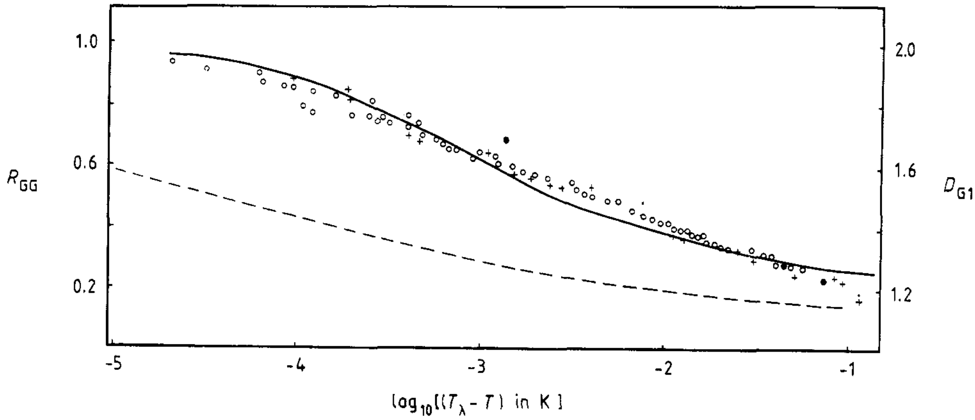

Figure 1. The reflection coefficient R_{GG} (left ordinate) and transmission coefficient D_{G1} (right ordinate) versus the logarithm of $T_\lambda - T$. The circles and the crosses are the experimental data of WB [1], the broken curve shows the results of the theory in [2], and the full curve represents the numerical results of equations (5.7) and (5.8).

The parameters π', L, Z_2, u_G, ρ_{Go} and h_{Go} are defined in [2] and references therein. The discussion given above suggests that

$$K_1 = a_1|\tau|^{-0.34} + b_1 \tag{5.12}$$

$$K_2 = a_2|\tau|^{-0.66} + b_2|\tau|^{-0.34} \tag{5.13}$$

where a_1, b_1, a_2 and b_2 are factors that cannot be determined from the theory due to the fact that the model that we used is a phenomenological model. However, we can determine them by considering that when $|\tau| \gtrsim 10^{-1}$ the WB theory [2] is correct and when $|\tau| < 10^{-1}$ the theory should be modified as far as possible. We find that if we take the K_1 and K_2 to be

$$K_1 = K_1^\circ(1 + 0.1|\tau|^{-0.34}) \tag{5.14}$$

$$K_2 = 5(|\tau|^{-0.66} - |\tau|^{-0.34}) \tag{5.15}$$

the theory will agree semi-quantitatively with the experimental data. In (5.14) $K_1^\circ = \frac{16}{9}(m/2\pi k_B T_o)^{1/2}$ is the value of K_1 outside the critical region.

In figures 1 and 2 the numerical results of equations (5.7)–(5.9) are shown and a comparison between both the experimental data and the uncorrected theoretical results of WB are given.

6. Summary and conclusions

In the present paper, by using the renormalisation group method we have studied the semi-infinite model E and have derived the critical behaviour of the transport coefficients concerning with the free surface of superfluid helium. With these results we have given a phenomenological correction to the theory of WB in the critical region, and suggested a qualitative explanation for the results of their sound conversion experiment.

The results of this paper is only a proposed one. First, the model that we used is not the exact model for the superfluid transition. A more realistic model, called model F [4] is required to give a quantitative description of the dynamics of the superfluid transition.

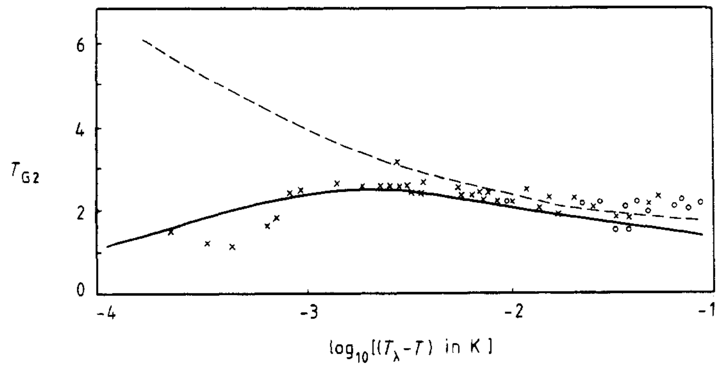

Figure 2. The conversion coefficient T_{G2} versus the logarithm of $T_\lambda - T$. The crosses and the circles are the experimental data of *WB* [1]. The broken curve shows the results of the theory of WB [2], and the full curve shows the numerical results of equation (5.9).

Secondly, for the more accurate description of the critical behaviour of the transport coefficients, a non-asymptotic treatment of the RG flow equations is necessary [11, 12], since κ_b, κ_s and κ_{bs} are not observable. Thirdly, and most importantly, the assumptions that we made in § 5 still need to be backed up by experiment and the parameters a_1, b_1, a_2 and b_2 should be calculated using a more exact model or obtained from experimental measurements.

Lastly, we point out that from figure 2 one can see that when $|\tau| < 10^{-3}$ the behaviour of T_{G2} cannot be described by equation (5.9) even qualitatively. This deviation may mean that some mechanism in the process still remains unconsidered. More exact studies in the future are desirable.

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